Optimal Scheduling of Port Operations
—Integrated Berth Allocation and Quay Crane Scheduling

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Outline

Introduction of the Integrated Problem BA-QCS
- Discrete berth allocation (BA)
- Quay crane scheduling (QCS)
- Integrated discrete BA and QCS

Part 1: Discrete Berth Allocation
- An MILP model by Lee and Wang (2010)
- A compact MILP model for BA
- A model based on multiple time grids

Part 2: Quay Crane Scheduling
- Consider crane travel times or not
- An MILP by Lee and Chen (2010)
- An MILP model considering travel times

Numerical Experiments (Three Examples)

Conclusion and Further Work
Discrete BA & QCSP

- **Q berths**
- **S ships**
- **M cranes**
- **N bays**

Berth allocation

Quay crane scheduling
Integrated Procedure of BA-QCS

- a number of ships: $S$
- arrival time: $a_s$
- number of bays: $N$
- bay process time: $p_i$
- quay cranes: $M$
- ready time: $br_u$
- a number of berths: $Q$

$H_{su}$: handling time of ship $s$ at berth $u$, $H_{su} = \left(\frac{c_{max}}{60}\right)_{su}$
A Schedule for Berth Allocation

60 ships are handled by 6 berths, $CV_{max} = 186$ hours
A ship with 16 bays is served by 5 cranes, $C_{max} = 665$ min
Part 1: Discrete Berth Allocation

Indices

$s, t =$ different ships
$u, v =$ different berths
$n =$ different time points

Sets

$V =$ a set of ships to be handled (with $S$ ships)
$U =$ a set of berths (with $Q$ berths)
$T =$ a set of time points on each unit

Parameters

$S = |V|$, the number of ships
$Q = |U|$, the number of berths
$H_{su} =$ handling time of ship $s$ at berth $u$
$a_s =$ ship arrival time, the time at which ship $s$ can start its handling
$br_u =$ berth ready time, the time at which berth $u$ get ready for handling ships
$B =$ a big number, for “Big M” constraints
Binary Variables:

\[ X_{su} = \] binary variables equal to 1, if and only if ship \( s \) is assigned to berth \( u \)

\[ Y_{st} = \] binary variables equal to 1, if and only if the handling completion time of ship \( s \) is no later than the handling start time of ship \( t \)

\[ Z_{st} = \] the handling of ship \( t \) follows the handling of ship \( s \), both are handled at the same berth

\[ X_{s,u,n} = \] ship \( s \) is assigned to time point \( n \) at berth \( u \)

Positive Continuous Variables:

\[ CV_s = \] handling completion time of ship \( s \)

\[ m_s = \] mooring or berthing time of container ship \( s \), which is handling start time of ship \( s \)

\[ ST_{u,n} = \] start of time point \( n \) at berth \( u \)

\[ CV_{\text{max}} = \text{makespan}, \quad CV_{\text{max}} = \max \{ CV_1, CV_2, \ldots, CV_S \} \]
Bi-Index Formulation by Lee and Wang (2010)

**BIF-Lee&Wang (Model I):** with three “Big M” constraints

Variables: $X_{su}$, $Y_{st}$, $m_s$, $CV_s$ and $CV_{max}$

Model constraints:

1. $CV_{max} \geq CV_s, \quad \forall s \in V$  
2. $CV_s = m_s + \sum_{u \in U} X_{su} \cdot H_{su}, \quad \forall s \in V$  
3. $\sum_{u \in U} X_{su} = 1, \quad \forall s \in V$

4. $CV_s - m_t + B \cdot Y_{st} \geq 0, \quad \forall s, t \in V$
5. $CV_s - m_t - B \cdot (1 - Y_{st}) \leq 0, \quad \forall s, t \in V$
6. $B \cdot [(X_{su} - 1) + (X_{tu} - 1)] \leq Y_{st} + Y_{ts} - 1, \quad \forall s, t \in V, \quad i < j; \quad \forall u \in U$
7. $m_s \geq a_s, \quad \forall s \in V$
8. $m_s \geq b_{ru} \cdot X_{su}, \quad \forall u \in U, \quad \forall s \in V$

**Objective function:**

$\text{Min} \quad CV_{\text{max}}$
Compact Bi-Index Formulation

CBIF (Model II): with two “Big M” constraints

Variables: $X_{su}$, $Z_{st}$, $CV_s$ and $CV_{max}$

Model constraints:

\[ \sum_{u \in U} X_{su} = 1, \quad \forall s \in V \]  \hspace{1cm} (10)

\[ X_{su} + \sum_{v \in U, v \neq u} X_{sv} + Z_{st} \leq 2, \quad \forall u \in U, \quad s = \{1, \ldots, S-1\}, t = \{s+1, \ldots, S\} \]  \hspace{1cm} (11)

\[ CV_t - CV_s + B \cdot (3 - Z_{st} - X_{su} - X_{sn}) \geq H_{tu}, \quad \forall u \in U, \quad s = \{1, \ldots, S-1\}, t = \{s+1, \ldots, S\} \]  \hspace{1cm} (12)

\[ CV_s - CP_t + B \cdot (2 + Z_{st} - X_{su} - X_{sn}) \geq H_{su}, \quad \forall u \in U, \quad s = \{1, \ldots, S-1\}, t = \{s+1, \ldots, S\} \]  \hspace{1cm} (13)

\[ CV_s - X_{su} \cdot H_{su} \geq X_{su} \cdot br_u, \quad \forall u \in U, \quad \forall s \in V \]  \hspace{1cm} (14)

\[ CV_s - \sum_{u \in U} X_{su} \cdot H_{su} \geq a_s \cdot \sum_{u \in U} X_{su}, \quad \forall s \in V \]  \hspace{1cm} (15)

\[ CV_{max} \geq CV_s, \quad \forall s \in V \]  \hspace{1cm} (16)

Objective function:

\[ \text{Min} \quad CV_{max} \]  \hspace{1cm} (17)
Multiple-Time-Grid Continuous-Time Formulation based on Castro et al., 2006

**MTGF(III):** Each berth uses a time grid with $|T|$ time start points to avoid the use of “Big M” constraints.

**Variables:** $X_{s,u,n}$, $ST_{u,n}$, and $CV_{\text{max}}$

**Constraints:**

\[ \sum_{u \in U} \sum_{n \in T} X_{s,u,n} = 1, \quad \forall s \in V \]  

\[ ST_{u,n+1} \geq ST_{u,n} + \sum_{s \in V} X_{s,u,n} \cdot H_{su}, \quad \forall u \in U, \forall n \in T, n \neq |T| \]  

\[ ST_{u,n} \geq br_u \cdot \sum_{s \in V} X_{s,u,n}, \quad \forall u \in U, \forall n \in T \]  

\[ ST_{u,n} \geq ar_s \cdot \sum_{s \in V} X_{s,u,n}, \quad \forall u \in U, \forall n \in T \]  

\[ CV_{\text{max}} \geq ST_{u,n} + \sum_{s \in I} X_{s,u,n} \cdot H_{su}, \quad \forall u \in U, \forall n \in T, n \neq |T| \]  

**Objective function:**

\[ \text{Min} \quad CV_{\text{max}} \]
Part 2: Quay Crane Scheduling

Indices

\( i, j, h = \text{different bays} \)
\( k, l = \text{different cranes} \)

Sets

\( I = \text{a set of bays to be processed (with } N \text{ bays)} \)
\( W = \text{a set of cranes (with } M \text{ cranes)} \)

Parameters

\( N = |I|, \text{ the number of bays} \)
\( M = |W|, \text{ the number of cranes} \)
\( p_i = \text{processing time of bay } i \)
\( c_{ij} = \text{crane travel time changing from bay } i \text{ to bay } j \)
\( o_{ri} = \text{bay ready time, the time at which bay } i \text{ can start its processing} \)
\( ur_k = \text{crane ready time, the time at which crane } k \text{ can get ready} \)
\( B = \text{a big number, for “Big M” constraints} \)
Quay Crane Scheduling (cont.)

**Binary Variables:**

- $X_{ik}$ = binary variables equal to 1, if and only if bay $i$ is assigned to crane $k$;
- $Y_{ij}$ = binary variables equal to 1, if and only if bay $i$ completes no later than bay $j$ starts, that is, $C_i < C_j - p_j$;
- $S_{ik}$ = bay $i$ is the first bay assigned to crane $k$, i.e. bay $i$ is the starting bay on crane $k$
- $Z_{ij}$ = bay $j$ follows bay $i$, both bay $i$ and bay $j$ are processed by the same crane

**Positive Continuous Variables:**

- $C_i$ = completion time of bay $i$
- $ST_i$ = processing start time of bay $i$
- $PST_i$ = processing start time of bay $i$
- $TST_i$ = processing start time of bay $i$
- $C_{max}$ = makespan, $C_{max} = \max\{C_1, C_2 \ldots, C_N\}$
Bi-index Formulation by Lee and Chen (2010)

**BIF-Lee&Chen (I):** Non-crossing, NOT consider $c_{ij}$, $o_i$, $u_{r_k}$

Variables: $X_{ik}$, $Y_{ij}$, $ST_i$ and $C_{max}$

Model constraints:

\[
C_{\text{max}} \geq ST_i + p_i, \quad \forall i \in I
\]

\[
ST_0 + p_0 = \sum_{j \in I} p_j, \quad ST_{N+1} + p_{N+1} = \sum_{j \in I} p_j
\]

\[
\sum_{k=0}^{M+1} X_{ik} = 1, \quad \forall 0 \leq i \leq (N+1)
\]

\[
X_{00} = 1, \quad X_{N+1,M+1} = 1
\]

\[
ST_i + p_i - ST_j + B^*Y_{ij} \geq 0, \quad \forall 0 \leq i, j \leq (N+1)
\]

\[
ST_i + p_i - ST_j - B^*(1-Y_{ij}) \leq 0, \quad \forall 0 \leq i, j \leq (N+1)
\]

\[
B^*(Y_{ij} + Y_{ji}) \geq \sum_{k=0}^{M+1} k^*X_{ik} - \sum_{l=0}^{M+1} l^*X_{jl} + 1, \quad \forall 0 \leq i < j \leq (N+1)
\]

\[
B^*(Y_{ij} + Y_{ji}) \geq \sum_{l=0}^{M+1} l^*X_{jl} - \sum_{k=0}^{M+1} k^*X_{ik} + (i-j), \quad \forall 0 \leq i < j \leq (N+1)
\]

Objective function:

\[
\text{Min} \quad C_{\text{max}}
\]
Compact Bi-Index Formulation for QCS (1/2)

CBIF-QCS (II): consider travel times $c_{ij}$, ready times $o_{ri}$, $u_{rk}$

Variables: $X_{ik}$, $Z_{ij}$, $Y_{ij}$, $PST_i$, $TST_i$ and $C_{max}$

Constraints:

1. $X_{ik} + \sum_{l \in W, l \neq k} X_{jl} + Z_{ij} \leq 2, \forall k \in W, i = \{1, \cdots, N-1\}, j = \{i+1, \cdots, N\}$ (31)

2. $PST_j - PST_i + B \cdot (3 - Z_{ij} - X_{ik} - X_{jk}) \geq p_i + c_{ij}$, $\forall k \in W, i = \{1, \cdots, N-1\}, j = \{i+1, \cdots, N\}$ (32)

3. $PST_i - PST_j + B \cdot (2 + Z_{ij} - X_{ik} - X_{jk}) \geq p_j + c_{ji}$, $\forall k \in W, i = \{1, \cdots, N-1\}, j = \{i+1, \cdots, N\}$ (33)

4. $PST_i \geq u_{rk} \cdot X_{ik}, \forall k \in W, \forall i \in I$ (34)

5. $PST_i \geq o_{ri} \cdot \sum_{k \in W} X_{ik}, \forall i \in I$ (35)

6. $C_{max} \geq PST_i + \sum_{k \in W} X_{ik} \cdot p_i, \forall i \in I$ (36)

7. $\sum_{k=0}^{M+1} X_{ik} = 1, \forall 0 \leq i \leq (N + 1)$ (37)
CBIF-QCS (II): non-crossing constraints

\[ PST_0 + p_0 = \sum_{j \in I} p_j, \quad PST_{N+1} + p_{N+1} = \sum_{j \in I} p_j \]  

\[ X_{00} = 1, \quad X_{N+1,M+1} = 1 \]  

\[ PST_i = TST_i + \sum_{\ell=0}^{N+1} (Z_{i\ell} \times c_{i\ell}), \quad \forall 0 \leq i \leq (N + 1) \]  

\[ PST_i + p_i - TST_j + B \times Y_{ij} \geq 0, \quad \forall 0 \leq i, j \leq (N + 1) \]  

\[ PST_i + p_i - TST_j - B \times (1 - Y_{ij}) \leq 0, \quad \forall 0 \leq i, j \leq (N + 1) \]  

\[ B \times (Y_{ij} + Y_{ji}) \geq \sum_{k=0}^{M+1} k \times X_{ik} - \sum_{l=0}^{M+1} l \times X_{jl} + 1, \quad \forall 0 \leq i < j \leq (N + 1) \]  

\[ B \times (Y_{ij} + Y_{ji}) \geq \sum_{l=0}^{M+1} l \times X_{jl} - \sum_{k=0}^{M+1} k \times X_{ik} + (i - j), \quad \forall 0 \leq i < j \leq (N + 1) \]  

Objective function:

\[ \text{Min} \quad C_{\text{max}} \]
Numerical Experiments (1/3)

- Formulated with \textit{GAMS 23.3}, solved by \textit{Cplex 12.1}
- Tested on a PC with i5-4570 CPU @ 3.20GHz & 8GB M.
- \textbf{Example 1 of BA}: $H_{su} \in [5, 25)$, $a_s \in [0, 25)$, $br_u \in [0, 25)$, \textit{in hours}.
- A busy port, up to 60 ships are handled by 6 berths in a week (168 h).

<table>
<thead>
<tr>
<th>Instance</th>
<th>BIF – Lee&amp;Wang (I)</th>
<th>CBIF (II)</th>
<th>MTGF (III)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>$S/Q$</td>
<td>$CV_{\text{max}}$</td>
<td>$CPU$ (Gap)</td>
</tr>
<tr>
<td>1</td>
<td>10/3</td>
<td>56*</td>
<td>25.59</td>
</tr>
<tr>
<td>2</td>
<td>11/3</td>
<td>58*</td>
<td>124.16</td>
</tr>
<tr>
<td>3</td>
<td>12/3</td>
<td>61*</td>
<td>659.24</td>
</tr>
<tr>
<td>4</td>
<td>13/3</td>
<td>68</td>
<td>7200 (7.35%)</td>
</tr>
<tr>
<td>5</td>
<td>14/3</td>
<td>68</td>
<td>7200 (5.88%)</td>
</tr>
<tr>
<td>6</td>
<td>15/3</td>
<td>71</td>
<td>7200 (26.76%)</td>
</tr>
<tr>
<td>7</td>
<td>20/2</td>
<td>150</td>
<td>7200 (58.11%)</td>
</tr>
<tr>
<td>8</td>
<td>30/3</td>
<td>147</td>
<td>7200 (70.69%)</td>
</tr>
<tr>
<td>10</td>
<td>40/4</td>
<td>/</td>
<td>7200 (no sltn)</td>
</tr>
<tr>
<td>11</td>
<td>50/5</td>
<td>/</td>
<td>7200 (no sltn)</td>
</tr>
<tr>
<td>12</td>
<td>60/6</td>
<td>/</td>
<td>7200 (no sltn)</td>
</tr>
</tbody>
</table>

* optimum
Numerical Experiments (2/3)

Example 2 of BA: $H_{su} \in [5, 25)$, $a_s \in [0, 168)$, $b_{ru} \in [0, 25)$

<table>
<thead>
<tr>
<th>Instance</th>
<th>BIF –Lee&amp;Wang (I)</th>
<th>CBIF (II)</th>
<th>MTGF (III)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>S/Q</td>
<td>$CV_{max}$</td>
<td>CPU (Gap)</td>
</tr>
<tr>
<td>1</td>
<td>20/2</td>
<td>187*</td>
<td>6.12</td>
</tr>
<tr>
<td>2</td>
<td>30/3</td>
<td>172*</td>
<td>23.50</td>
</tr>
<tr>
<td>3</td>
<td>40/4</td>
<td>179*</td>
<td>2995.71</td>
</tr>
<tr>
<td>4</td>
<td>50/5</td>
<td>181</td>
<td>7200 (2.21%)</td>
</tr>
<tr>
<td>5</td>
<td>60/6</td>
<td>180</td>
<td>7200 (1.67%)</td>
</tr>
</tbody>
</table>

Berth (Not so busy as before) $CV_{max} = 177.00$

\[ S_{min} = 0.00 \]

$\theta_{ij} = 177.00$ (Not so busy as before)
Numerical Experiments (3/3)

An Example of QCS: $p_i \in [30, 180)$, $c_{ij} = |j - i|$ (in minutes)

<table>
<thead>
<tr>
<th>Instance</th>
<th>BIF –Lee&amp;Chen (I)</th>
<th>CBIF-QCS (II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>$\max C$</td>
<td>$CPU (\text{Gap})$</td>
</tr>
<tr>
<td>1</td>
<td>10/2</td>
<td>539*</td>
</tr>
<tr>
<td>2</td>
<td>11/3</td>
<td>405*</td>
</tr>
<tr>
<td>3</td>
<td>12/3</td>
<td>458*</td>
</tr>
<tr>
<td>4</td>
<td>13/3</td>
<td>492*</td>
</tr>
<tr>
<td>5</td>
<td>14/3</td>
<td>534</td>
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<tr>
<td>6</td>
<td>15/3</td>
<td>545</td>
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<tr>
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<td>20/4</td>
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<td>539</td>
</tr>
<tr>
<td>10</td>
<td>30/5</td>
<td>647</td>
</tr>
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<table>
<thead>
<tr>
<th>Crane</th>
<th>Bay</th>
<th>Start Time</th>
<th>End Time</th>
<th>Process Time</th>
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<tbody>
<tr>
<td>k1</td>
<td>i1</td>
<td>436</td>
<td>594</td>
<td>158</td>
</tr>
<tr>
<td>k1</td>
<td>i2</td>
<td>305</td>
<td>435</td>
<td>130</td>
</tr>
<tr>
<td>k1</td>
<td>i3</td>
<td>187</td>
<td>304</td>
<td>117</td>
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<td>k1</td>
<td>i4</td>
<td>114</td>
<td>186</td>
<td>72</td>
</tr>
<tr>
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<td>i8</td>
<td>0</td>
<td>110</td>
<td>110</td>
</tr>
<tr>
<td>k2</td>
<td>i5</td>
<td>317</td>
<td>496</td>
<td>179</td>
</tr>
<tr>
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<tr>
<td>k2</td>
<td>i9</td>
<td>217</td>
<td>308</td>
<td>91</td>
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<tr>
<td>k2</td>
<td>i12</td>
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<td>214</td>
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<td>k3</td>
<td>i7</td>
<td>494</td>
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<td>k3</td>
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<td>i19</td>
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<tr>
<td>k4</td>
<td>i20</td>
<td>0</td>
<td>140</td>
<td>140</td>
</tr>
</tbody>
</table>

A QC schedule
Conclusion and Further Work

- We have developed mixed integer programming (MIP) models for the discrete BA-QSCP.
- The proposed QCS model considers crane travel times as well as non-crossing constraints.
- The proposed BA model (Model III) based on multiple time grids is able to avoid the “Big M” constraints, and solve industrial-size instances (up to 60 ships and 6 berths) to optimality or near-optimality in 2h.

- In fact, we also have developed meta-heuristic methods to solve far larger instances (e.g., 200 ships and 16 berths) in short computational time, but cannot prove the optimality.
- We will further develop branch-and-price algorithms based on column generation, which can be much faster than MIP and prove the optimality.
- If a port is quite busy, we should apply continuous BA-QCS models which can make full use of the berth space and quay cranes, more challenging.
Thank You!

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